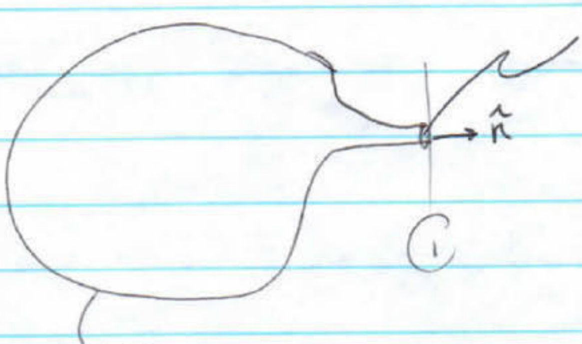
(M) example

(6)

Ex| Tank with $\forall = 0.05\ m^3$ with air
$\rho = 800$ kPa abs
$T = 15$ C

At $t=0$ air leaks out at speed $|\underline{V}| = 311$ m/s
exit density $\rho = 6.13\ kg/m^3$ with $A = 65\ mm^3$.



Find $\frac{d\rho}{dt}$ in tank.

$N = M_{ass}$ and $\eta = 1$ Version (1)

$$\frac{D}{Dt} M_{sys} = 0 = \frac{d}{dt} \iiint_{CV} \rho \eta \, d\forall + \iint_{CS} \rho \eta \, \underline{V}_{s/b} \cdot \hat{n} \, dA$$

$$0 = \frac{d}{dt} \rho \iiint_{CV} dV + \iint \rho \, \underline{V}_{s/b} \cdot \hat{n} \, dA$$

$$0 = \frac{d}{dt}(\rho \forall_{tank}) + (\rho V A)\Big|_{1}$$

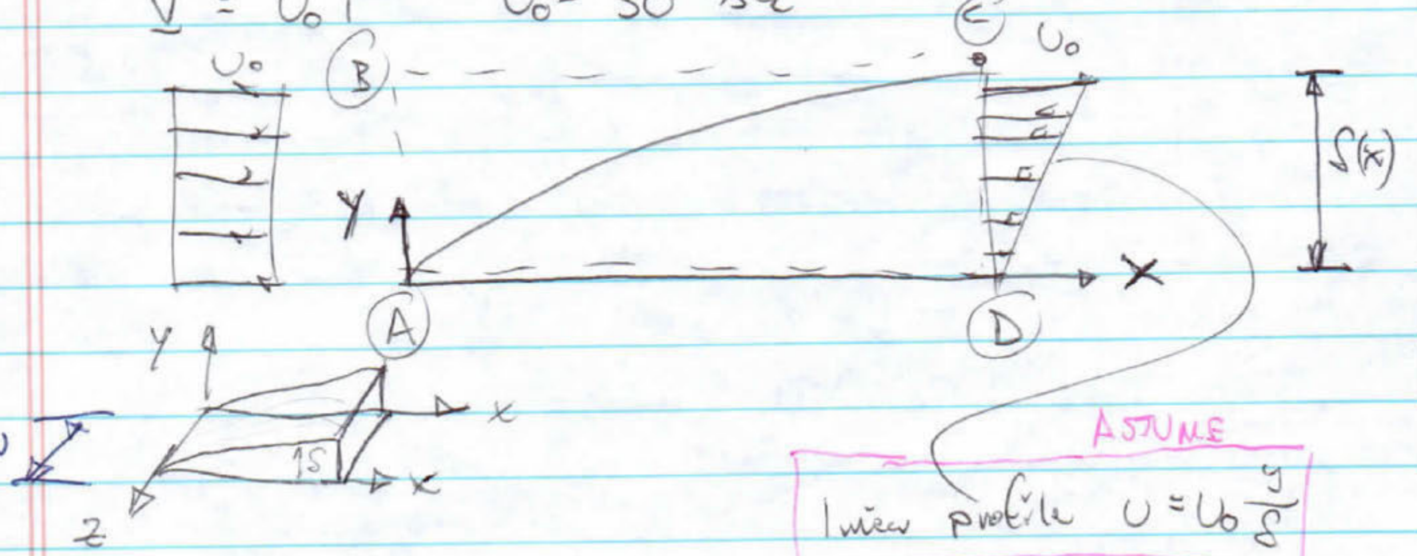
so $\forall_{tank} \frac{d\rho}{dt} = -(\rho V A)\Big|_{1}$

$$\frac{d\rho}{dt} = \frac{-(\rho V A)\big|_{1}}{\forall} = \dots = -2.48 \ \frac{kg/m^3}{sec}$$

(m) example

Ex

$\underline{V} = U_0 \hat{i} \qquad U_0 = 30 \text{ m/sec}$



ASSUME

linear profile $u = U_0 \dfrac{y}{\delta}$

Assume: $\delta = 5 \text{ mm}$

Plate width $= w = 0.6 \text{ m}$ (in z-direction)

$$(m) \quad \frac{DM_{sys}}{Dt} = 0 = \cancel{\frac{d}{dt}\iiint_{cv} \rho \, d\forall} + \iint_{cs} \rho \, \underline{V}_{s/b} \cdot \hat{n} \, dA$$

Version (1)

$\cancel{D}$ 0 for steady flow

$$\therefore \iint_{cs} \rho \underline{V} \cdot \hat{n} \, dA = 0$$

$$\underset{AB}{\iint_{(1)}} + \iint_{BC} + \iint_{CD} + \cancel{\iint_{AD}} = 0$$

$$\iint_{BC} = -\iint_{AB} - \iint_{CD}$$

$$\iint_{BC} \rho \underline{V} \cdot \hat{n}\, dA = -\iint_{AB} \rho \underline{V} \cdot \hat{n}\, dA - \iint_{CD} \rho (\underline{V} \cdot \hat{n})\, dA$$

$$= +\rho U_o \delta w - \iint \rho \left(U_o \frac{y}{\delta}\right) dy\, dz$$

$$= \rho U_o \delta w - \int_{y=0}^{\delta} w \rho U_o \frac{y}{\delta}\, dy$$

$$= \rho U_o \delta w - \rho U_o w \frac{\delta}{2}$$

$$= \frac{\rho U_o \delta w}{2}$$

$$= \dot{m} = 0.0558 \text{ kg/sec}$$

$\boxed{\eta = \underline{V}_{s/o}}$ specific momentum

(1)

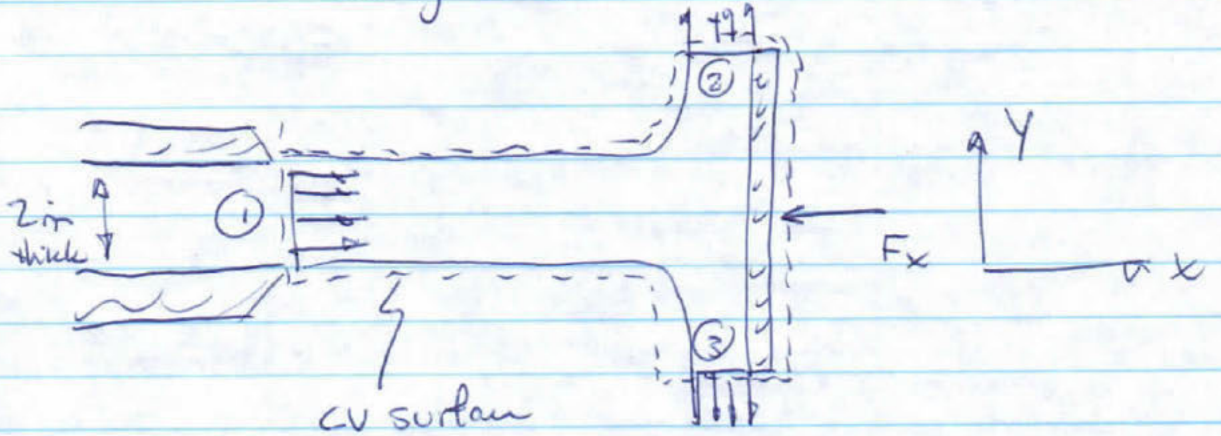
Ex1

$$\Sigma \underline{F} = \frac{D}{Dt} \iiint_{sys} \rho \underline{V}_{s/o}\, d\forall = \frac{d}{dt} \iiint_{CV} \rho \underline{V}_{s/o}\, d\forall + \iint_{CS} \rho \underline{V}_{s/o} (\underline{V}_{s/b} \cdot \hat{n})\, dA$$

The $\boxed{x\text{-component}}$ becomes

Version (1)

$$\Sigma F_x = \underbrace{\frac{d}{dt} \iiint_{CV} \rho V_x\, d\forall}_{\substack{=0 \text{ for} \\ \text{steady state}}} + \iint_{CS} \rho V_x (\underline{V}_{s/b} \cdot \hat{n})\, dA$$



A plane jet of water 2 in thick hits a fixed vertical plate. The flow rate out of the jet is $\boxed{10\ ft^3/s}$ per foot of width (in z-direction).

Calc the force F_x (per foot of width) to hold plate in place.

$$Q = \text{flow rate} = \iint_{1} V_x\, dA = V_{x1} A_1$$

so $$V_{x1} = \frac{Q_1}{A_1} = (10\ ft^3/s) / (\tfrac{2}{12} ft \times 1\ ft) = 60\ ft/s$$

Bernoulli's eqn (in an inertial ref frame)

$$\frac{V_1^2}{2} + \frac{P_1}{\rho_1} + gh_1 = \frac{V_2^2}{2} + \frac{P_2}{\rho_2} + gh_2 = \text{¢} \qquad h = \text{height}$$

Since $h_1 \approx h_2 \simeq h_3$ + $P_2 = P_3 = P_1$ we are left with

$$V_1 = V_2 = V_3 = 60\ \text{f/s}$$

So applying x-mom says

$$\Sigma F_x = -F_p = \iint_{CS} \rho V_x (\underset{S/B}{V} \circ \hat{n})\, dA$$

$$= \iint_{A_1} \rho V_{1x} (+V_{1x}\hat{i} \circ (-\hat{i}))\, dA + \iint_{A_2} \rho \underset{0}{\cancel{V_{2x}}} (V_2 \hat{j} \circ \hat{j})\, dA$$

$$+ \iint_{A_3} \rho \underset{0}{\cancel{V_{3x}}} (-V_3 \hat{j} \circ -\hat{j})\, dA$$

or

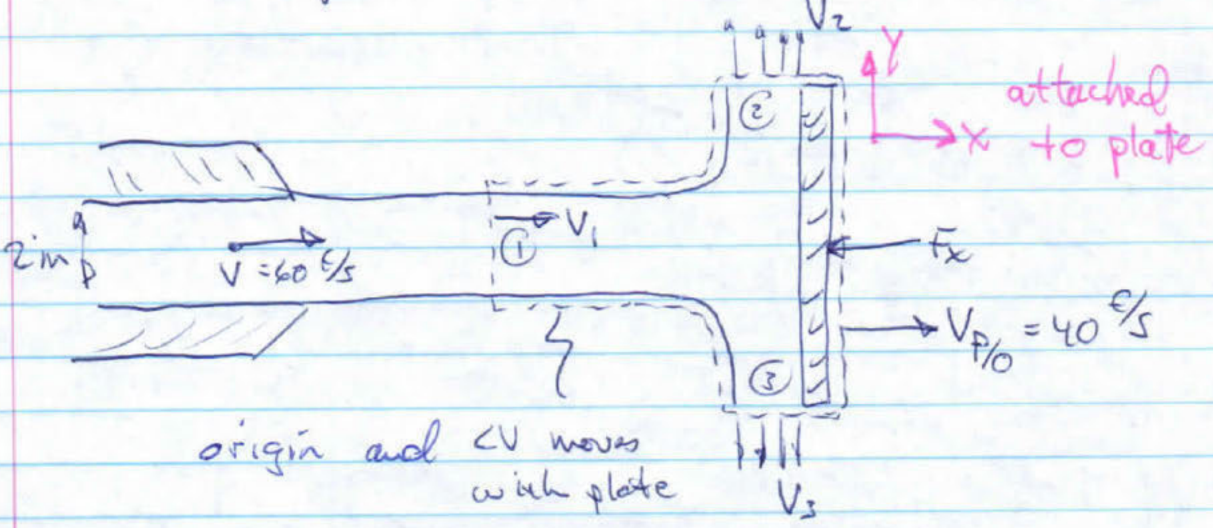
$$-F_p = \iint_{A_1} \rho V_1 (-V_1)\, dA = -\rho V_1^2 A_1$$

$$F_p = \rho_\omega V_1^2 A_1 \qquad V_1 = 60\ \text{f/s}, \quad A_1 = \tfrac{2}{12}\,\text{ft} \times 1\,\text{ft}$$

$= 1164$ lbf/ft of width

//

Ex) Now s'pose the plate is moving to the right at 40 ft/s. Calc the force on the plate and the velocity at the upper edge relative to an observer on the ground.



$$\underline{V}_{w/p} = \underline{V}_{w/o} + \underline{V}_{o/p} = \underline{V}_{w/o} - \underline{V}_{p/o}$$

so in x-dir

$$ {}_xV_{w/p} = {}_xV_{w/o} - {}_xV_{p/o} = 60\text{ f/s} - 40\text{ f/s} = 20\text{ f/s}$$

So (x-mom) becomes

$$\Sigma F_x = \iint_{CS} \rho V_{x\,f/cv} \left(\underline{V}_{f/cv} \cdot \hat{n}\right) dA$$

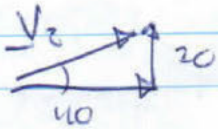
$$-F_x = \iint_{①} \rho\, {}_xV_{w/p,1} \left(-{}_xV_{w/p,1}\right) dA = -\rho V_{w/p,1}^2 A_1$$

$$F_x = \rho V_{w/p,1}^2 A_1 \qquad V_{w/p,1} = 20\text{ f/s}$$

$$= \dots = 129.3 \text{ lbf/ft of width}$$

Note that to someone on the ground would see

$$\underline{V}_2 = 40\,\text{ft/s}\,\hat{i} + 20\,\text{ft/s}\,\hat{j}$$



so

$\theta =$